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Borelli Center

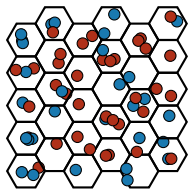
École Normale Supérieure de Paris-Saclay

June 16, 2026

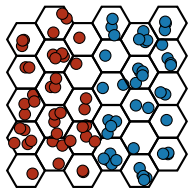
Analysing Spatial Heterogeneity

using Optimal Transport and applied to electoral results

in collaboration with Joseph Touzet (ÉNS Paris-Saclay)



Weak Heterogeneity



High Heterogeneity

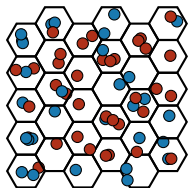
Heterogeneity/Segregation Measures

Goals

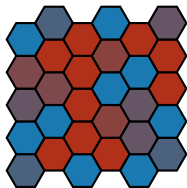
- Propose indices to measure the **spatial heterogeneity** of the **red** and **blue** distributions: are they well mixed or not?
- Indices of segregation/heterogeneity are powerful tools for summarising the spatial relationships between population groups and thereby providing the basis for analysis and public policy intervention.

Various area of applications

- An idea arising from research on residential segregation (Du Bois & Eaton 1899, Massey & Denton 1988, 1993)...
- since extended to many other fields: medical care access (Launay and al. 2019, Lucas-Gabrielli Mangeney 2019), education (Hogrebe and al. 2021), economic seasonality concentration (Lo Magno and al. 2017)...



Weak Heterogeneity



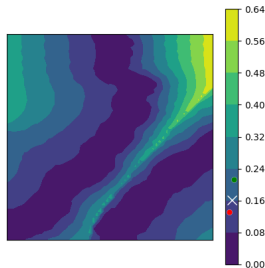
Local aspatial measure

- › Spatial vs. aspatial: does the index depend on the relative positions of the areal units one to another?
- › Global: a score summarizing a whole situation.
- › Local: lower scale measure.
- › Massey Denton 1988 (The Dimensions of Residential Segregation): evenness, exposure, clustering, concentration, centralization.
- › An index cannot measure all these dimensions.

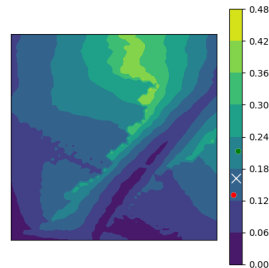
Dimensions of heterogeneity



a Distribution of the two populations (red and green), with constant population density and without boundary conditions. The two regions are separated by a random border generated using Perlin noise.



b Concave-cost local heterogeneity index



c Convex-cost local heterogeneity index

Figure: Synthetic example based on a noisy border generated with Perlin noise.

- Paris is divided into I units, each corresponding to a unique polling station.
- We are interested in the score of each candidate.

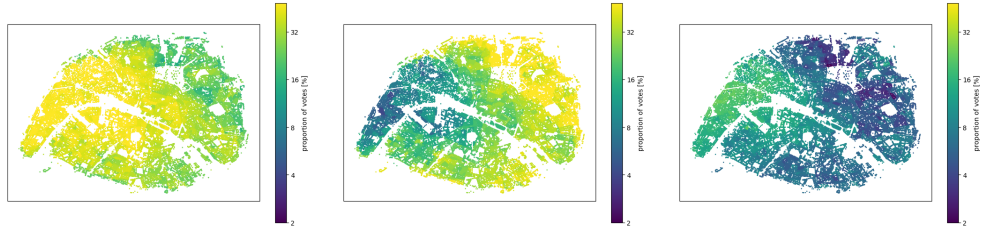
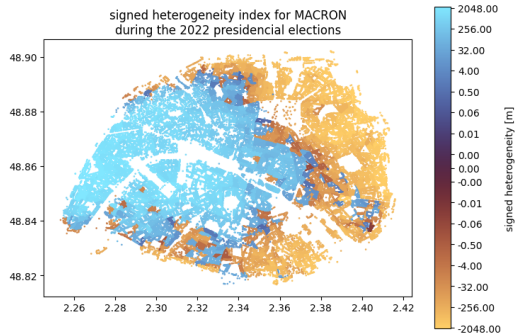


Figure: First round results of the 2022 French presidential election. Results in Paris for Macron, Mélenchon and Zemmour

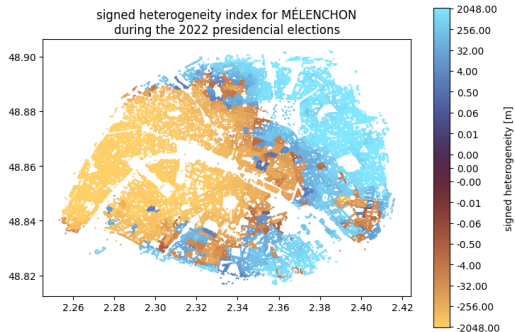
Touzet and B. 2025: simulations



- French 2022 Presidential results of 4 candidates (among 12) (*Paris area*):

Candidate	OT-Index (global)	Score (%)
Macron	519 meters	35,49
Mélenchon	980 meters	29,93
Zemmour	1005 meters	8,2
Le Pen	295 meters	5,39

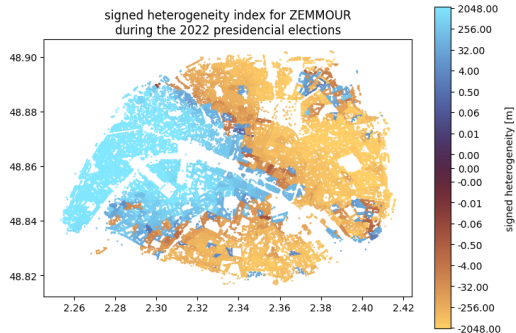
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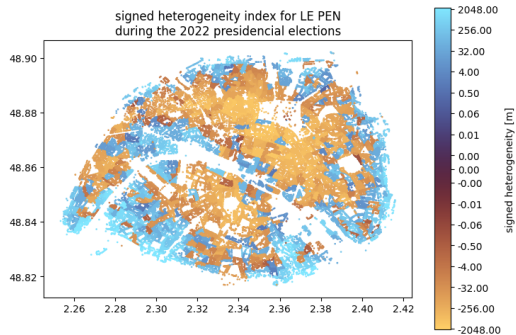
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Optimal Transport (OT) to investigate spatial heterogeneity

Optimal Transport in a Nutshell: Find the best way to match a spatial distribution (e.g. votes distribution) to a benchmark. The lower the score, the better the matching.

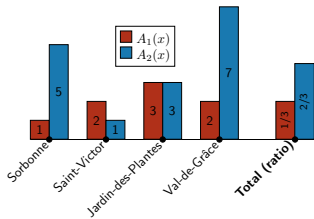
Model: fix a candidate n .

- We have A_n the votes' distribution for this candidate. We construct an ideal distribution of votes $\bar{A}_n$ corresponding to the homogeneous distribution of the same number of votes.
- We use Optimal Transport method to match the distribution A_i to $\bar{A}_i$.
- Optimal Transport = Nearest Neighbor + Incompressibility (not enough space) = Allocation problem

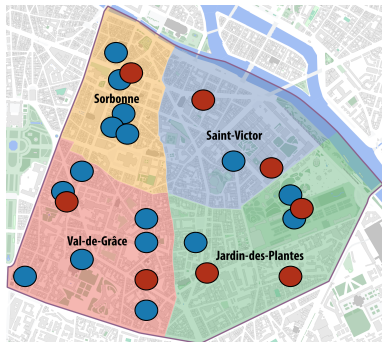
Bibliography:

- Articles using OT to study segregation/heterogeneity: Jakubs (1981), Lo Magno and al. (2017), Morgan (1983) Ferrante and al. (2020), Peng & Murphy (2025).

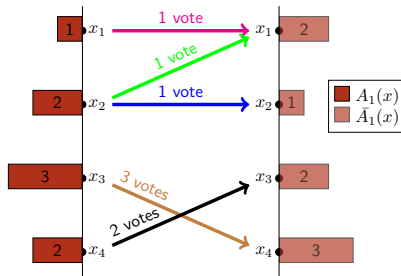
Method's explanation



$$\gamma_n = \frac{1}{8} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$



Simulated election in Paris' Latin Quarter

An example of transference plan γ_n

- › Readily-to-interpret: average distance to travel to reach a general practitioner.
- › Global: a score on the general situation, easy to compare year to year variation.
- › Local: heat map.
- › Geographical index: spatial arrangement of areal units is taken into account.